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EQUATIONS OF MOTION FOR FLIGHT SIMULATION OF THE ASROC MISSILE

by

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ABSTRACT

Equations of motion are presented for a three dimensional trajectory simulation under the assumption that, during the thrust phase, the configuration is a rigid body and has 90 degree rotational symmetry. For flight simulation during the after-burning phase equations based on particle ballistic theory are presented. The instantaneous position of the missile in space is defined relative to a spherical, rotating earth. Aerodynamic coefficients considered during the thrust phase are functions of Mach number, angle of attack and effective roll angle. During the after-burning phase the aerodynamic drag coefficient is a function of Mach number. The formulation was designed for the specific purpose of generating accurate fire control data for the ASROC missile but may have wide application among other types of rockets. For this reason certain terms have been included in the model which are not significant for the ASROC application. Among them are terms to simulate effects of a rotating earth. As an option the free-flight after-burning phase may be simulated by a six degree of freedom model.

INTRODUCTION

A mathematical model having six degrees of freedom, three for translational and three for angular motion, is required for accurate simulation of the flight of the ASROC missile during the thrust phase, while a particle mathematical model simulates with sufficient accuracy the motion after burning. Current theory regarding the aerodynamic response of ASROC, as it emerges from a closed cell launcher into the air stream, specifies that the aerodynamic moments build up to peak values, rather than instantaneously, during the first few milliseconds after launch. This effect has been included in the model.

At motor separation a net forward thrust is exerted on the second stage configuration producing disturbances, including a significant but predictable instantaneous increase in the velocity of the second stage. Separation of the airframe from the payload later in the trajectory also causes disturbances to the motion of the payload. In addition, if the payload is a torpedo it is decelerated to a safe water entry velocity by a parachute. (ASROC trajectory events are illustrated in Figure 1.)

The mathematical model described in this report simulates (1) motion on a closed cell, non-tip-off launcher using a one-dimensional differential equation of motion wherein only thrust and gravity are considered, (2) motion during the thrust phase using equations of motion having six degrees of freedom with the added provision that the components of the restoring moment coefficient generated by the fins be time-dependent for a short period after launch, and (3) motion during the after-burning phase using equations of motion based on particle ballistics. Disturbances accompanying transition from the powered to the after burning phase and from the separation of the airframe and the subsequent deployment of the parachute are considered. The mathematical model generates trajectory elements for each stage, adjusts the terminal position, velocity and time to account for the disturbances, and employs the adjusted values as initial conditions for the succeeding stage.

When the special features incorporated for accurate simulation of the trajectory for the ASROC missile are removed, the model reduces essentially to those described in reference 1 as the particle model and the Littlejohn model. The equations are coded, in Fortran, for solution on either an IBM 7090 or STRETCH computer using the Runge-Kutta procedure for numerical integration.

REFERENCE FRAMES

a. Reference Axes and Orientation Angles

Two orthogonal coordinate systems are used to define the missile's position and angular orientation for rigid body simulation during the thrust phase. The earth fixed reference system (x,y,z) is a right-handed coordinate system with x in a horizontal plane pointing down range; y vertical, positive upward, and z perpendicular to the xy plane. The position of the missile's center of gravity is defined relative to this system during the powered and the after-burning phases.

The (X,Y,Z) reference system is also an orthogonal right-handed coordinate system having its origin at the center of gravity of the missile. The X axis is coincident with the missile's longitudinal axis, positive forward; the Y axis is in the vertical plane containing the X axis; the Z axis is perpendicular to the XY plane.

Orientation angles θ and ψ are used to define the instantaneous elevation and azimuth angles, respectively, of the missile with respect to the inertial (x,y,z) reference frame as shown in Figure 2. θ is the angle between the missile's longitudinal axis, X, and the horizontal (xz) plane and ψ is the azimuth angle of the missile, measured in the horizontal plane, relative to the x axis. θ is positive about the positive Z axis, ψ is positive about the negative y axis.

The total velocity vector with respect to the ground, $\vec{V}$, is computed from components u, v, and w which are components of the velocity of the origin of the X,Y,Z axes relative to the x,y,z system. u, v, and w are measured in the direction of X, Y, and Z, respectively.

b. Transformation of Reference Frames

Velocity components of the center of gravity u, v, and w (along the X,Y,Z axes) are expressed relative to the inertial (x,y,z) reference system through use of the transpose of the transformation matrix A where

$$A = \begin{pmatrix} \cos\theta\cos\psi & \sin\theta & \cos\theta\sin\psi \\ -\sin\theta\cos\psi & \cos\theta & -\sin\theta\sin\psi \\ -\sin\psi & 0 & \cos\psi \end{pmatrix} .$$

ASROC TRAJECTORY EVENTS

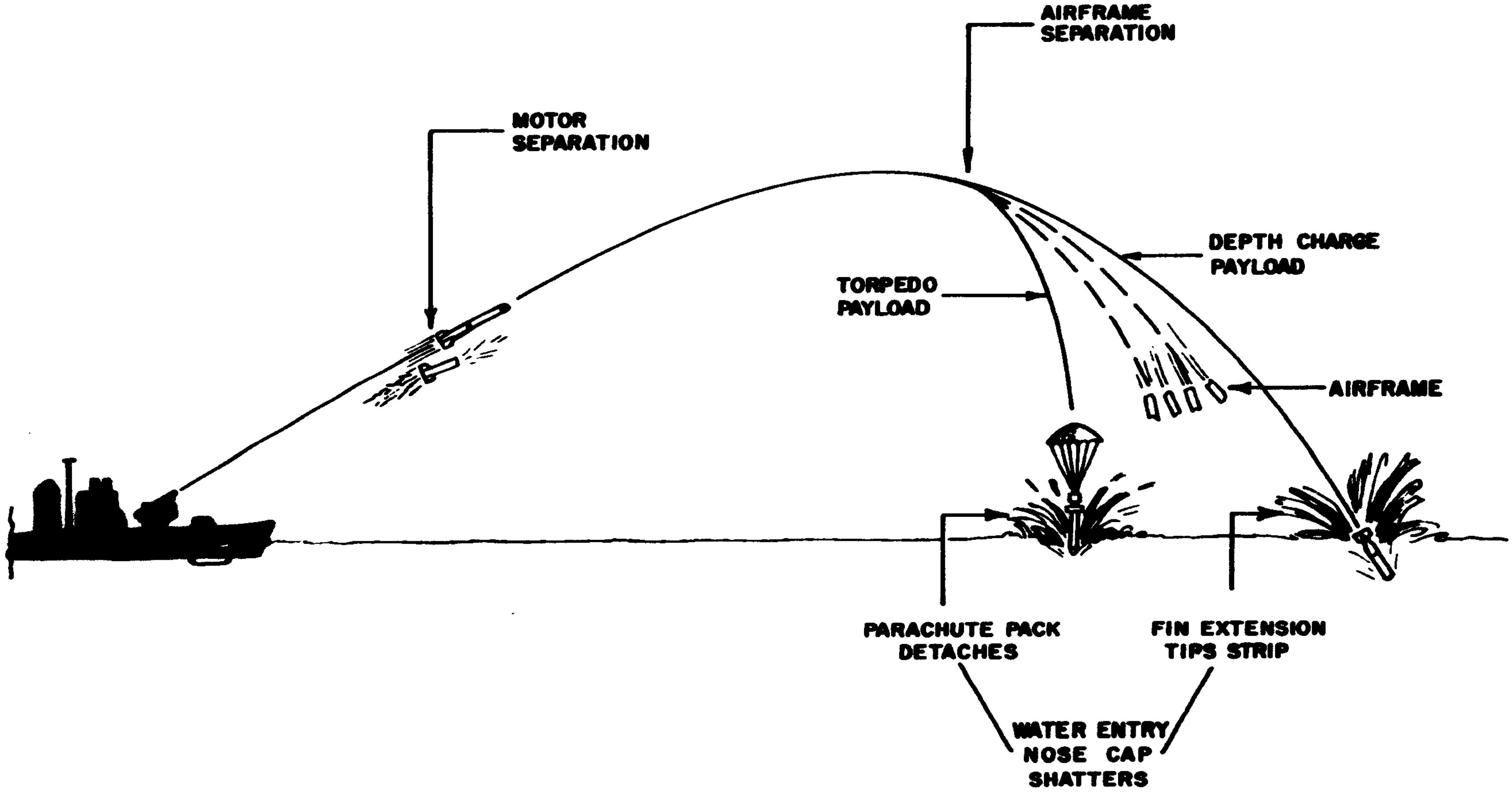


FIGURE I

COORDINATE SYSTEMS, ORIENTATION ANGLES
AND THE MISSILES SIX DEGREES OF FREEDOM

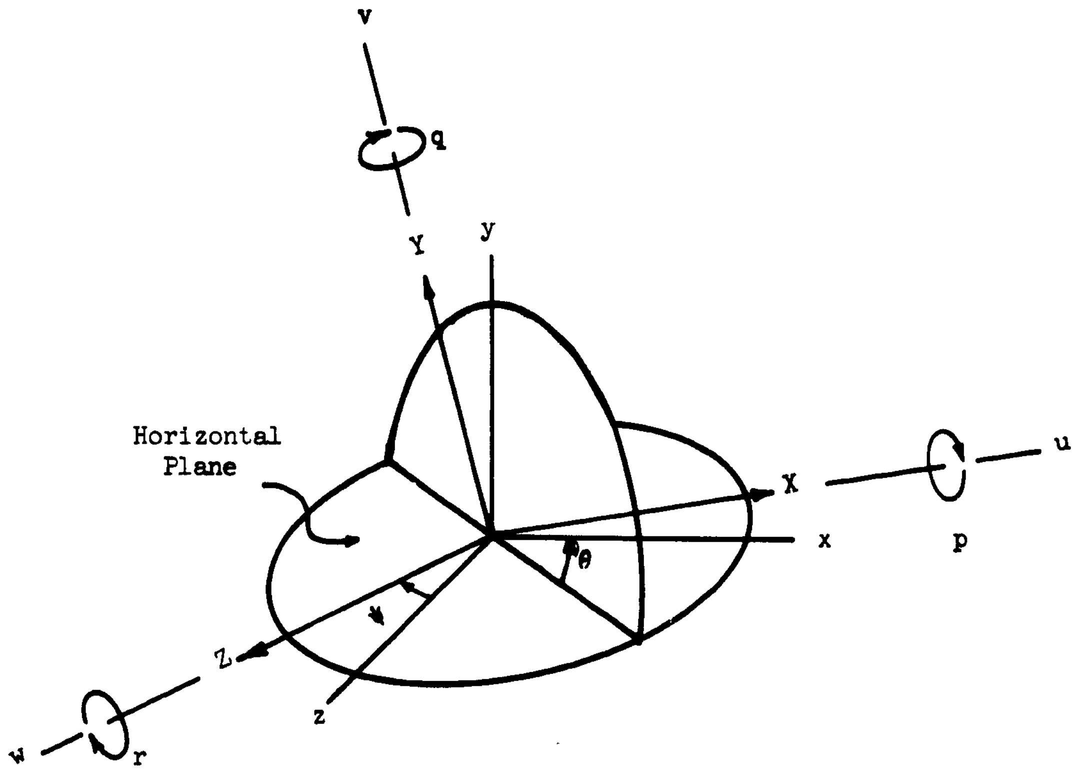


Figure 2

Then

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} A^* \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

where A^* is the transpose of the matrix A.

c. Angular Velocity of the Reference Frame

The total angular velocity vector of the missile, $\vec{\omega}$, is comprised of components p, q, and r measured about the X,Y,Z axes, respectively, (see Figure 2). Because the X,Y,Z system does not spin with the missile (coincident with the constraint imposed on the Z axis to lie in the horizontal, xz, plane), the angular velocity vector of the missile reference frame is different from the angular velocity vector of the missile. It is shown in Appendix B that the angular velocity of the XYZ reference system presented in the inertial reference system has the components

$$(q \tan \theta, -q/\cos \theta, r)$$

about the X, Y, and Z axes, respectively. The angular velocity vector of the missile may then be written as

$$\begin{pmatrix} \dot{\phi} \\ \dot{\psi} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} p - q \tan \theta \\ -q/\cos \theta \\ r \end{pmatrix}$$

where ϕ is the roll angle of missile about the X axis with respect to the vertical plane containing the launching lugs.

RIGID BODY PHYSICAL PARAMETERS

Mass, moments of inertia (axial and transverse), and the center of gravity position are simulated as time dependent variables using the following equations:

a. Mass

$$m = m_o + \int_{t_o}^{t} \dot{m}\, dt \quad , \quad \dot{m} = -\frac{T}{V_g}$$

where m is the instantaneous mass, m_o the initial mass, $\dot{m}$ the rate at which the propellant is expended, T the instantaneous thrust, and V_g the effective gas velocity. V_g may be expressed as the product of the specific impulse (I_{sp}) and gravity.

b. Axial Moment of Inertia

$$I_X = I_{XB} + k^2 m_p$$

where I_X is the instantaneous axial moment of inertia, I_{XB} the axial moment of inertia with all propellant expended, k^2 the square of the radius of gyration of the missile about the longitudinal axis (assumed to be constant), and m_p ($= m_{po} + \int_{t_o}^{t} \dot{m}\, dt$), the instantaneous mass of the propellant.

c. Transverse Moment of Inertia

$$I_Y = \alpha_2 + k_p^2 m - \alpha_1 \lambda_g$$

where I_Y is the instantaneous transverse moment of inertia, λ_g the instantaneous center of gravity position with respect to the missile's nose, k_p^2 the square of the radius of gyration of the propellant about a transverse axis through the propellant center of gravity and α_1 and α_2 are constants. The method used to determine the constants α_1, α_2, and k_p^2 is given in section d. below.

d. Center of Gravity Position

$$\lambda_g = \lambda_p + \frac{\alpha_1}{m}$$

where λ_g is the instantaneous center of gravity position with respect to the nose; λ_p is the center of gravity position of the propellant (assumed constant).

The procedure for determining the constants α_1, α_2 and k_p^2 for the ASROC missile is as follows:

1. Obtain measurements of the center of gravity position, λ_g, for the fully loaded (unburnt) round for the round with all propellant expended. Solve the system of linear equations

$$\lambda_{g,o} = \lambda_p + \frac{\alpha_1}{m_o}$$

$$\lambda_{g,b} = \lambda_p + \frac{\alpha_1}{m_b}$$

for α_1 and λ_p.

2. Obtain measurements of the transverse moment of inertia for the fully loaded (unburnt) round and for the round with all propellant expended. Solve the system of linear equations

$$I_{y,o} = \alpha_2 + k_p^2\, m_o - \alpha_1\, \lambda_{g,o}$$

$$I_{y,b} = \alpha_2 + k_p^2\, m_b - \alpha_1\, \lambda_{g,b}$$

for α_2 and k_p^2 .

EQUATIONS OF MOTION

Components of the total translational and angular acceleration vectors measured relative to the X,Y,Z reference frame are, respectively:

$$\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \frac{1}{m}(\vec{F}_T + \vec{F}_A) - \begin{pmatrix} q \tan\theta \\ q \\ r \end{pmatrix} x \begin{pmatrix} u \\ v \\ w \end{pmatrix} + A \begin{pmatrix} o \\ g_{approx} \\ o \end{pmatrix} - 2A\left[\vec{\Omega} \; x \; \vec{V}\right]$$

$$\begin{pmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{pmatrix} = \frac{1}{I_y}\left[(\vec{M}_A + \vec{M}_T) - \begin{pmatrix} q \tan\theta \\ q \\ r \end{pmatrix} x \begin{pmatrix} I_x \; p \\ I_y \; q \\ I_y \; r \end{pmatrix}\right]$$

where

$\vec{F}_T$ = force due to thrust

$\vec{F}_A$ = sum of the aerodynamic forces

$\vec{g}$ = gravitational acceleration vector

$\left[\vec{\Omega} \; x \; \vec{V}\right]$ = acceleration vector caused by the earth's rotation

$\vec{M}_A$ = sum of the torques produced by aerodynamic moments (including aerodynamic trim)

$\vec{M}_T$ = $(\vec{M}_T^* + \vec{M}_J)$, sum of the torques produced by thrust (including jet damping torque, $\vec{M}_J$)

A differential equation representing the gravity free acceleration measured by an accelerometer during the thrust phase is also computed. It is assumed that the accelerometer is oriented along the X axis and is therefore sensitive only to accelerations along X. The equation is

$$\dot{V}_{accel} = \frac{1}{m}\left[T \cos\epsilon + QSC_X(M)\right] - (q^2 + r^2)(\lambda_g - \lambda_{accel})$$

λ_{accel} is the distance, along the longitudinal axis of the missile, between the nose of the missile and the accelerometer.

a. Forces and Moments

Aerodynamic Forces ($\vec{F}_A$)

Aerodynamic Forces considered are:

Force	Coefficient
Drag	$C_X(M,\alpha)$
Normal Force - sum of the roll and nonroll dependent components	$C_Z(M,\alpha) = C_{Z0}(M,\alpha) + C_{Z8}(M,\alpha) \sin 4 (\Lambda + \phi)$
Magnus	$C_{yp}(M,\alpha)$

Λ is the dihedral angle between the XZ plane and the yaw plane and ϕ is the dihedral angle between the XY plane and a plane containing the missile longitudinal axis and the launching lugs. These angles are shown in Figure 3.

Mach number and angle of attack are computed as follows:

Mach Number:

$$M = \frac{\sqrt{T_{ssl}}}{A_{ssl}} \cdot \frac{V_a}{\sqrt{T^*(y)}}$$

Angle of attack:

$$\alpha = \tan^{-1} \frac{\sqrt{v_a^2 + w_a^2}}{|u_a|}$$

$$\alpha_{tr} = \tan^{-1} \frac{\sqrt{(v_a + u_a \tan \delta \cos \phi_{tr})^2 + (w_a + u_a \tan \delta \sin \phi_{tr})^2}}{|u_a|}$$

where

T_{ssl} = temperature of the air at sea level under standard atmosphere conditions

$T^*(y)$ = temperature of the air at altitude y

A_{ssl} = speed of sound at sea level under standard atmospheric conditions

V_a = magnitude of the missile velocity vector relative to the air

u_a, v_a, w_a = components of the missile's total velocity vector relative to the air measured along the X,Y,Z reference axes, respectively

1. Thrust Forces ($\vec{F}_T$)

The mathematical model considers thrust forces along all three missile fixed axes, i.e., forces resulting from a thrust vector not coincident with the missile longitudinal axis are included. The following examples are given to illustrate the types of malalignment which may be simulated.

Example 1. Linear Thrust Malalignment: The nozzle axis is eccentric with respect to the missile X axis by the distance r_N, in a direction defined by the angle ϕ_N.

Example 2. Angular Thrust Malalignment: The axis normal to the nozzle plate is malaligned by the angle ϵ with respect to the missile X axis in a plane oriented by the angle ϕ_c with respect to the body fixed Y axis. The axis of thrust passes through the center of gravity of the round.

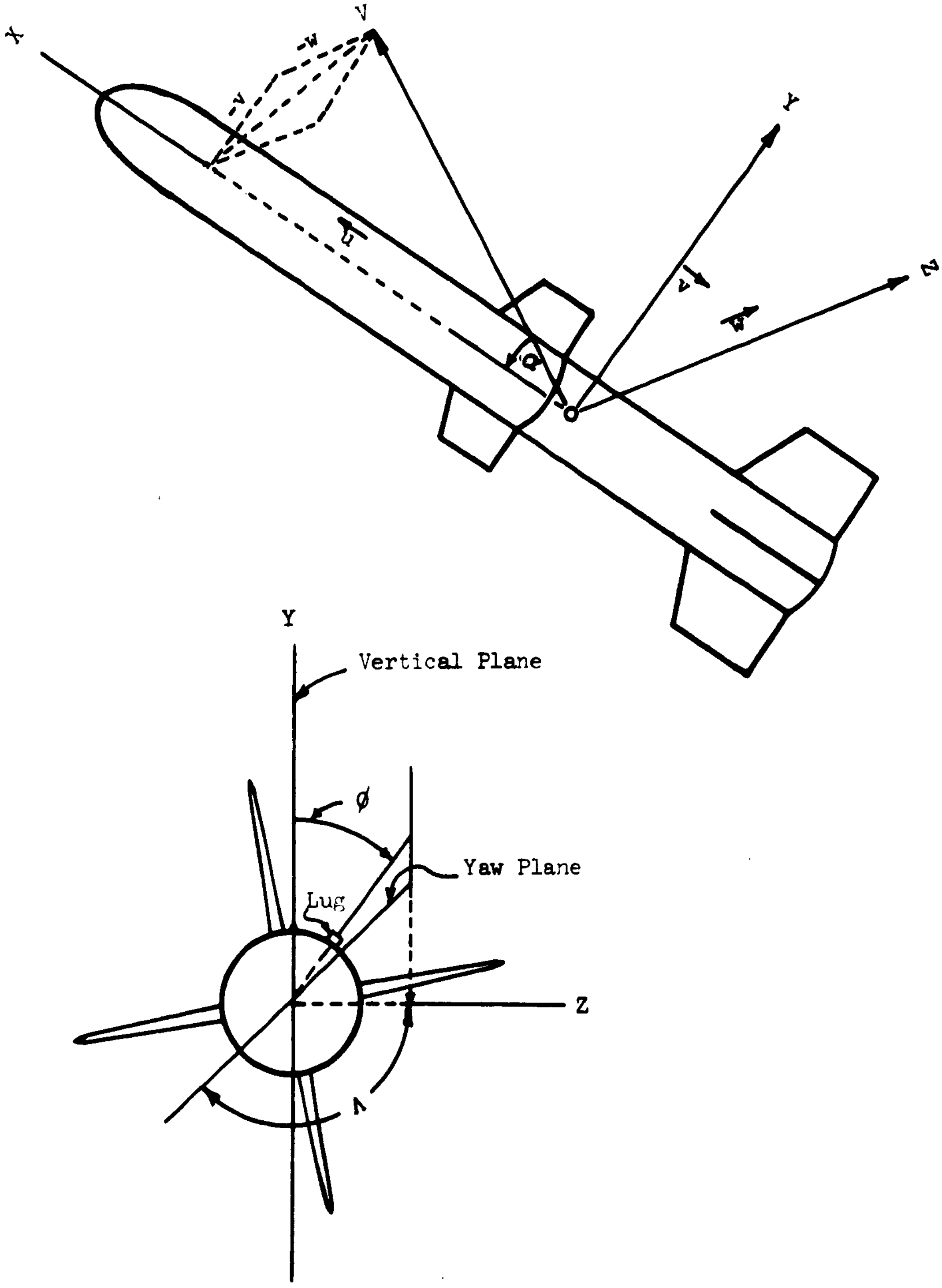
Example 3. Linear and Angular Thrust Malalignment: A combination of examples 1 and 2.

(The moments generated about the body fixed axes due to thrust malalignment are discussed in the section entitled "Thrust Moments.")

The forces acting along the missile fixed axes as a result of thrust malalignment are given by the equation

$$\vec{F}_T = \left[\phi_{th}\right] \cdot \vec{T}$$

DIAGRAM SHOWING α, Λ, AND ϕ



Looking forward along X

Figure 3

where

$$\vec{T} = T \begin{pmatrix} \cos \epsilon \\ \sin \epsilon \cos \phi_c \\ \sin \epsilon \sin \phi_c \end{pmatrix}$$

and

$$\phi_{th} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \phi_{th} & -\sin \phi_{th} \\ 0 & \sin \phi_{th} & \cos \phi_{th} \end{pmatrix}$$

ϵ = the angle between the missile longitudinal axis and the thrust vector

T = thrust

ϕ_{th} = $\phi_{th,o} + \phi$, the instantaneous dihedral angle between the vertical plane containing the missile Y axis and the plane containing the malaligned thrust vector and a vector parallel to the missile longitudinal axis

ϕ_c = the orientation, with respect to a vertical plane formed by the malaligned thrust vector and the missile longitudinal axis of the plane formed by the missile longitudinal axis and an axis through the nozzle center

2. Aerodynamic Moments ($\vec{M}_A$)

Aerodynamic moments considered are:

Moment	Coefficient
Restoring moment - sum of the body alone, booster fin and airframe fin components	$C_m = C_{mo}(M,\alpha) + C_{mof_1}(\alpha) + C_{mof_2}(\alpha) + \left[C_{msf_1}(\alpha) + C_{msf_2}(\alpha)\right] \cos 4(\Lambda + \phi)$
Side moment - sum of booster and airframe fin components	$C_n = \left[C_{n7f_1}(\alpha) + C_{n7f_2}(\alpha)\right] \sin 4(\Lambda + \phi)$

Magnus moment	$C_{np}(M,\alpha)$
Cross spin damping moment	$C_{mq}(M,\alpha)$
Axial spin moment	$C_{\ell\delta_A}(M)$
Axial spin damping moment	$C_{\ell p}(M)$

3. Thrust Moments ($\vec{M}_T^* = \vec{M}_T + \vec{M}_J$)

The jet damping moment ($\vec{M}_J$) results from an impingement of gases inside the thrust chamber which exerts a damping moment on the missile experiencing angular velocities. The effect is approximated by the terms

$$\vec{M}_J = \dot{m}\ell^2 \begin{pmatrix} 0 \\ q \\ r \end{pmatrix}$$

where ℓ is a length associated with jet damping.

Thrust moments ($\vec{M}_T$) generated as a result of a mal-aligned thrust vector are also considered. Considering the cases described in the section on thrust forces, the moments corresponding to these forces are given by the vector equation

$$\vec{M}_T = \left[\phi_{th}\right] \cdot \left[(\vec{r} \times \vec{T}) + \vec{M}_\kappa\right]$$

where

$$\vec{r} = \begin{pmatrix} -(L - \lambda_g) \\ r_N \cos \phi_N \\ r_N \sin \phi_N \end{pmatrix}$$

and

$$\vec{M}_{\varkappa} = \varkappa T \begin{pmatrix} \cos \epsilon \\ \sin \epsilon \cos \phi_c \\ \sin \epsilon \sin \phi_c \end{pmatrix}$$

L = length of the rocket

r_N = the perpendicular distance between the thrust vector and the missile longitudinal axis

ϕ_N = the orientation, with respect to a vertical plane containing the missile longitudinal axis, of the plane containing r_N

$\varkappa$ = thrust axial moment coefficient

$\varkappa T$ = the rolling torque about the thrust vector regardless of the alignment of the latter and will generate moments about the three body fixed axes when both ϵ and ϕ_c are other than zero

The development of terms to simulate a malaligned thrust vector are based on the assumption of a single nozzle. However, the equations apply to the multiple nozzle case provided that components of the thrust forces along the axes normal to the nozzle plate are known. The simpliest case would be one in which the nozzle centers are equidistant from the nozzle plate center, have the same cant angle, and are spaced symmetrically with respect to a reference plane containing the nozzle center.

4. Aerodynamic Trim

Deviations in the external configuration of the missile from the symmetrical shape to which basic wind tunnel aerodynamic data apply may result in the aerodynamic normal force and restoring moment being zero for a nonzero angle of attack. The magnitude of this angle of attack is referred to as the trim angle, δ. The axis defined by δ is the aerodynamic trim axis which specifies the direction of the airspeed vector for which there are no normal forces. The resulting cross velocity components with respect to the air under conditions of aerodynamic trim are

$$\begin{pmatrix} u_a \\ v_a \\ w_a \end{pmatrix}_{\delta \neq 0} = \begin{pmatrix} u_a \\ v_a \\ w_a \end{pmatrix}_{\delta = 0} + \begin{pmatrix} 0 \\ u_a \tan \delta \cos \phi_{tr} \\ u_a \tan \delta \sin \phi_{tr} \end{pmatrix}$$

The orientation angle ϕ_{tr} is the instantaneous dihedral angle about the missile X axis between the XY plane and the plane containing the aerodynamic trim axis and the missile X axis. At any instant

$$\phi_{tr} = \phi_{tr,o} + \phi$$

where the subscript "O" indicates initial value.

b. Gravity

The simulation of acceleration due to gravity is based on the inverse square law of force as applied to a spherical earth:

$$\vec{g} = - g_o \frac{r_o^2}{r^3} \vec{r}$$

where

$\vec{g}_o$ = the gravitational acceleration at the origin of the earth fixed reference system

r_o = the radius of the earth

$\vec{r}$ = the position vector of the missile center of gravity in space

r = $|\vec{r}|$

At the origin of the earth fixed (x,y,z) system the gravity vector is

$$\vec{g}_o = - g_o \left(\frac{1}{r_o}\right) \vec{r}_o = \begin{pmatrix} 0 \\ - g_o \\ 0 \end{pmatrix}$$

At any point on the trajectory, away from the origin, the gravity vector is approximated by

$$\vec{g}_{approx} = \vec{g}_o + \Delta\vec{g}_o$$

where

$\Delta\vec{g}_o$ = the derivative of $\vec{g}$ (with respect to position) evaluated at the origin multiplied by the change in position ($\Delta\vec{g}_o = \vec{\nabla}\vec{g}_o \, \Delta\vec{r}$)

For the present case, the z component of position is small enough to be neglected insofar as its influence on $\vec{g}$ is concerned. In the simulation, therefore, the following gravity vector is employed:

$$\vec{g}_{approx} = \vec{g}_o + \begin{pmatrix} \dfrac{-g_o x}{r_o} \\ \dfrac{2g_o y}{r_o} \\ 0 \end{pmatrix}$$

Note that elsewhere in the report the radius of the earth is denoted by r_e, r is the component of the total angular velocity measured about the Z axis and the subscript "0" denotes time of ignition of the propellant.

c. Coriolis Acceleration

The acceleration vector resulting from the assumption of a rotating earth presented in the (x,y,z) reference system, is

$$-2\left[\vec{\Omega} \times \vec{V}\right]$$

where $\vec{\Omega}$ is the angular velocity vector of the earth (directed along the polar axis) and $\vec{V}$ the total velocity vector of the missile relative to the earth fixed system. Expressions for the components of $\vec{\Omega}$ are derived as follows:

Define $\bar{L}$ as the angle between the earth's axis and the horizontal plane (positive in the northern hemisphere) and Az_L as the angle from north to the horizontal projection of the launcher or the azimuth angle of the positive x axis. The positive z axis will then have azimuth $90° + Az_L$. Therefore, a unit vector drawn horizontally northward will have x and z components which are, respectively cos Az_L and cos $(90° + Az_L) = -\sin Az_L$. The horizontal projection of $\vec{\Omega}$ points

northward and has length $\Omega \cos \bar{L}$, so its x and z components are $\Omega \cos L \cos Az_L$ and $-\Omega \cos \bar{L} \sin Az_L$, respectively. Its y component is $\Omega \sin \bar{L}$. The three components of $\vec{\Omega}$ may then be written as

$$|\vec{\Omega}| = \begin{pmatrix} \cos \bar{L} \cos Az_L \\ \sin \bar{L} \\ -\cos L \sin Az_L \end{pmatrix} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}$$

Then,

$$-2\left[\vec{\Omega} \times \vec{V}\right] = -2 \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} \times \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix}$$

Presentation of the vector in the missile fixed reference system is accomplished by premultiplying the above expression by the A matrix.

d. Atmosphere

Wind

Horizontal wind is expressed as a velocity, W, and an orientation, ψ_W, where

$$\psi_W = Az_W - Az_L$$

Az_W is the angle between north and the direction from which the wind blows. Az_L is the angle between north and the vertical plane through the launcher. Thus, ψ_W is the angle between the vertical plane containing the launcher line and the wind vector. Components of W expressed relative to the (x,y,z) axes are then given by

$$\begin{pmatrix} W_x \\ W_y \\ W_z \end{pmatrix} = -W \begin{pmatrix} \cos (Az_W - Az_L) \\ 0 \\ \sin (Az_W - Az_L) \end{pmatrix}$$

The computer program provides for the computation of the wind vector from components of ambient wind and from a moving launcher as follows:

$$\begin{pmatrix} W_x \\ W_y \\ W_z \end{pmatrix} = - W_A \begin{pmatrix} \cos\,(Az_{W_A} - Az_L) \\ 0 \\ \sin\,(Az_{W_A} - Az_L) \end{pmatrix} - W_c \begin{pmatrix} \cos\,(Az_T - Az_L) \\ 0 \\ \sin\,(Az_T - Az_L) \end{pmatrix}$$

where

W_A = magnitude of the ambient wind

Az_{W_A} = angle between north and the direction from which the wind blows

W_c = velocity of the launcher (or a moving cart on which the launcher is mounted)

Az_T = angle between north and the direction of motion of the launcher (or cart)

Since the aerodynamic forces and moments on the missile are functions of missile velocities relative to the air, the components (u_a, v_a, w_a) measured along the missile reference axes are computed as follows:

$$\begin{pmatrix} u_a \\ v_a \\ w_a \end{pmatrix} = \begin{pmatrix} u \\ v \\ w \end{pmatrix} - \begin{pmatrix} \\ A \\ \\ \end{pmatrix} \begin{pmatrix} W_x \\ W_y = 0 \\ W_z \end{pmatrix}$$

It follows that the total velocity relative to the air mass, V_a, is

$$V_a = (u_a^2 + v_a^2 + w_a^2)^{1/2}$$

The air density, ρ, and air temperature, T^*, are inputs as functions of altitude, y, and must be provided in the special format and size described in Appendix C.

Force, moment and acceleration terms contributing to the total translational and angular acceleration vectors are summarized in Table 1.

TABLE 1

FORCE AND MOMENT TERMS

Force, Moment or Acceleration	Absolute Scalar Magnitude	Vector Component Along or About X	Y	Z
Axial Drag	$QSC_x(M,\alpha)$	$QSC_x(M,\alpha)$	0	0
Normal Force (including force due to aerodynamic trim)	$QSC_z(M,\alpha)$	0	$QSC_z(M,\alpha)\ \frac{\tilde{\alpha}*}{\sqrt{\tilde{\alpha}^2+\tilde{\beta}^2}}$	$QSC_z(M,\alpha)\ \frac{\tilde{\beta}*}{\sqrt{\tilde{\alpha}^2+\tilde{\beta}^2}}$
Jet Thrust (force)	T	$T\cos\epsilon$	$T\sin\epsilon\cos(\phi_c+\phi_{th})$	$T\sin\epsilon\sin(\phi_c+\phi_{th})$
Gravitation and Coriolis	$m(g+\Delta g)$	$g\left[-m\sin\theta+\frac{1}{r_e}(x\cos\theta\cos\psi-2y\sin\theta)\right]$	$-g\left[m\cos\theta+\frac{1}{r_e}(x\sin\theta\cdot\cos\psi+2y\cos\theta)\right]$	$-mg\,x\sin\psi$
		$-\left[a_y\dot{z}-{}_z\dot{y}\right]\cos\theta\cos\psi$	$-(a_y\dot{z}-a_z\dot{y})\sin\theta\cos\psi$	$+\left[a_y\dot{z}-a_z\dot{y}\right]\sin\psi$
		$+\left[a_x\dot{z}-{}_z\dot{x}\right]\sin\theta$	$+(a_x\dot{z}-a_z\dot{x})\cos\theta$	$+\left[a_x\dot{y}-a_y\dot{x}\right]\cos\psi$
		$-\left[a_x\dot{y}-a_y\dot{x}\right]\cos\theta\sin\psi$	$-(a_x\dot{y}-a_y\dot{x})\sin\theta\sin\psi$	
Magnus Force	$QSC_{yp}(M,\alpha)\frac{pd}{2V_a}$	0	$QSC_{yp}(M,\alpha)\frac{pd}{2V_a}\ \frac{w_a}{\sqrt{v_a^2+w_a^2}}$	$-QSC_{yp}(M,\alpha)\frac{pd}{2V_a}\ \frac{v_a}{\sqrt{v_a^2+w_a^2}}$
Magnus Moment	$QSdC_{np}(M,\alpha)\frac{pd}{2V_a}$	0	$QSC_{np}(M,\alpha)\frac{pd}{2V_a}\ \frac{v_a}{\sqrt{v_a^2+w_a^2}}$	$QSC_{np}(M,\alpha)\frac{pd}{2V_a}\ \frac{w_a}{\sqrt{v_a^2+w_a^2}}$
Side Force	$QSC_{y\gamma}(\alpha)$*	0	$QSC_{y\gamma}(\alpha)\ \frac{v_a}{\sqrt{v_a^2+w_a^2}}$	$QSC_{y\gamma}(\alpha)\ \frac{w_a}{\sqrt{v_a^2+w_a^2}}$
Side Moment	$QSdC_{n\gamma}(\alpha)$*	0	$QSdC_{n\gamma}(\alpha)\ \frac{w_a}{\sqrt{v_a^2+w_a^2}}$	$QSdC_{n\gamma}(\alpha)\ \frac{v_a}{\sqrt{v_a^2+w_a^2}}$
Restoring (overturning) Moment	$QSdC_m(M,\alpha)$*	0	$QSdC_m(M,\alpha)\ \frac{\tilde{\beta}}{\sqrt{\tilde{\alpha}^2+\tilde{\beta}^2}}$	$-QSdC_m(M,\alpha)\ \frac{\tilde{\alpha}}{\sqrt{\tilde{\alpha}^2+\tilde{\beta}^2}}$

TABLE 1 (Continued)

Force, Moment or Acceleration	Absolute Scalar Magnitude	Vector Component Along or About: X	Y	Z
Cross-spin Damping Moment	$QSdC_{mq}(M,\alpha)\ \frac{d}{2V_a}\ (q^2 + r^2)^{1/2}$	0	$QSdC_{mq}(M,\alpha)\ \frac{qd}{2V_a}$	$QSdC_{mq}(M,\alpha)\ \frac{rd}{2V_a}$
Axial Moment (spin moment)	$QSdC_{\ell\delta_A}(M) \cdot \delta_A$	$QSdC_{\ell\delta_A}(M) \cdot \delta_A$	0	0
Axial Moment (spin damping)	$QSdC_{\ell p}(M)\ \frac{pd}{2V_a}$	$QSdC_{\ell p}(M)\ \frac{pd}{2V_a}$	0	0
Jet Damping (moment)	$\dot{m}\ \ell^2\ (q^2 + r^2)^{1/2}$	0	$\dot{m}\ \ell^2\ q$	$\dot{m}\ \ell^2\ r$
Jet Torque	$\left[\phi_{th}\right] \cdot \left\lvert(\vec{r}_T \times \vec{T}) + \vec{M}_\kappa\right\rvert$	$T\left[r_N \sin\epsilon \sin(\phi_c - \phi_N) + \kappa \cos\epsilon\right]$	$T\cos\phi_{th}\left[(L - \lambda_g)\sin\epsilon\sin\phi_c + r_N \sin\phi_N \cos\epsilon + \kappa\sin\epsilon\cos\phi_c\right] + T\sin\phi_{th}\left[(L - \lambda_g)\sin\epsilon\cos\phi_c + r_N\cos\epsilon\cos\phi_N + \kappa\sin\epsilon\sin\phi_c\right]$	$T\sin\phi_{th}\left[(L - \lambda_g)\sin\epsilon\sin\phi_c + r_N\cos\epsilon\sin\phi_N + \kappa\sin\epsilon\cos\phi_c\right] - T\cos\phi_{th}\left[(L - \lambda_g)\sin\epsilon\cos\phi_c + r_N\cos\epsilon\cos\phi_N + \kappa\sin\epsilon\sin\phi_c\right]$

$*\tilde{\alpha} = v_a + u_a \tan\delta \cos\phi_{tr}$

$\tilde{\delta} = w_a + u_a \tan\delta \sin\phi_{tr}$

$C_Z(M,\alpha) = C_{Z0}(M,\alpha) + C_{Zs}(M,\alpha) \sin 4\ (\phi + \Lambda)$

$C_m(M,\alpha) = C_{mo}(M,\alpha) + C_{mof1}(\alpha) + C_{mof2}(\alpha) + \left(C_{msf1}(\alpha) + C_{msf2}(\alpha)\right) \cos 4\ (\phi + \Lambda)$

$C_{n7}(\alpha) = \left(C_{n7f1}(\alpha) + C_{n7f2}(\alpha)\right) \sin 4\ (\phi + \Lambda)$

$C_{y7}(\alpha) = \left(C_{y7f1}(\alpha) + C_{y7f2}(\alpha)\right) \sin 4\ (\phi + \Lambda)$

INITIAL CONDITIONS AND STAGING CONSIDERATIONS

The initial conditions for the missile trajectory at launch presented in the inertial reference system are:

a. For the position vector, $\vec{R}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix}$$

where R is the length and θ the elevation angle of the launcher.

b. For the velocity vector, $\dot{\vec{R}}$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \dot{R} \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix}$$

where the vector $\dot{\vec{R}}$ is obtained by integrating the one dimensional differential equation

$$\ddot{R} = \frac{T}{m} - g \sin\theta$$

to the time when R equals the launcher length. The differential equation used for simulating the gravity free acceleration sensed by the accelerometer during launch is

$$\dot{V}_{accel} = \frac{T}{m}$$

FREE FLIGHT PARTICLE TRAJECTORY MODEL

Impact predictions based on a simulation of missile motion using a three dimensional particle model between thrust termination and impact were almost as accurate as would have been obtained using rigid body equations of motion. The above consideration and the resulting reduction in computer time make the particle model a logical choice for simulating the after burning phase.

In the particle model the total translational acceleration vector, expressed in component form relative to the inertial (x,y,z) reference system, is given by

$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} = \frac{1}{2m} \rho(y) S\, V_a\, C_{X2 \text{ or } 3}(M) \begin{pmatrix} \dot{x} - W_x \\ \dot{y} \\ \dot{z} - W_z \end{pmatrix} - \begin{pmatrix} g \frac{x}{r_e} + a_z \dot{y} - a_y \dot{z} \\ g\left(1 + 2 \frac{y}{r_e}\right) + a_x \dot{z} - a_z \dot{x} \\ a_x \dot{y} - a_y \dot{x} \end{pmatrix}$$

To start computations of the second stage of the trajectory, initial values of time, position and velocity, appropriate for the time at which maximum velocity is attained, must be generated from the terminal point computed using the six degree of freedom model. The procedure coded to obtain the initial values is as follows:

a. Compute the time maximum velocity is expected to occur by adjusting the time of motor separation by a correction, based on observed performance, supplied as input,

b. compute components of velocity appropriate at the time maximum velocity is expected to occur by adjusting values computed using the six degree of freedom model at time of motor separation by amounts, based on observed performance, supplied as inputs, and,

c. compute position coordinates appropriate for the time maximum velocity is expected to occur by adjusting the coordinates computed using the six degree of freedom model at motor separation by an amount approximated using the assumption of gravity free travel in a vacuum.

The equations coded to perform these functions are:

a. <u>Time</u>

$$t_{V_{max}} = t_{co} + \Delta t_{1-2}$$

where $t_{V_{max}}$ is the time maximum velocity is attained and t_{co} is the motor separation time. Δt_{1-2} is an input constant and represents the average time between separation of the motor from the payload and the time of maximum velocity of the payload.

b. Velocity

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix}_{t_{V_{max}}} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix}_{t_{co}} + \begin{pmatrix} \Delta\dot{x} \\ \Delta\dot{y} \\ 0 \end{pmatrix}$$

where $\Delta\dot{x}$ and $\Delta\dot{y}$ are the x and y components of velocity resulting from the impulsive force imparted to the second stage at motor separation. $\Delta\dot{x}$ and $\Delta\dot{y}$ are evaluated as functions of the expected velocity at motor separation and supplied as inputs. Mass and second stage diameter are the only other new inputs required.

c. Position Coordinates

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{t_{V_{max}}} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{t_{co}} + \Delta t_{1-2} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix}_{t_{co}}$$

Motion between the time that separation of the airframe is initiated and the time when the parachute is fully deployed is not simulated. The initial values of time, velocity, and position necessary to "restart" the trajectory at time of full parachute deployment are obtained by adjusting values generated at time of airframe separation as follows:

a. Compute estimate of time required for the parachute to become fully deployed, and adjust the time of airframe separation by this estimate.

b. Compute position coordinates appropriate for adjusted time by applying estimates, based on the assumption of gravity free travel between the time of airframe separation and the adjusted time, computed as in a. above. It is assumed that there is no loss in velocity during parachute deployment. (If the configuration is a depth charge, the parachute is not used. It is assumed that separation of the airframe is instantaneous and that no disturbances are imparted to the payload.)

The equations coded to perform these functions are:

a. Time

$$t_{3,0} = t_2 + \Delta t_{2-3}$$

where

$t_{3,0}$ = time when the parachute is fully deployed.

t_2 = time that separation of the airframe is initiated.

$\Delta t_{2-3} = C_1 + C_2 V_{t_2} + C_3 (V_{t_2})^2$, scaled value, based on observed performance, of the time required for full deployment of the parachute. (C_1, C_2 and $C_3 = 0$ when flight of the depth charge is simulated.)

V_{t_2} = missile velocity at time of airframe separation.

b. Position Coordinates

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{3,0} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}_2 + \Delta t_{2-3} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix}_2$$

Complete descriptions of separation phenomena and disturbances during separation of the airframe and the subsequent deployment of the parachute are given in reference 2.

REFERENCES

1. NPG Conf Report No. 1530, NAVORD Report No. 5136, Preparation of Exterior Ballistic Tables, dated 14 Mar 1958
2. NWL Conf Report No. 1742, The Ballistic Analysis and Preparation of Fire Control Data for the ASROC Missile, dated 24 Mar 1961

APPENDIX A

GLOSSARY

(A)	transformation matrix relating the missile axis reference system to the earth-fixed reference system
a_{ssl}	standard sea level value for the speed of sound
Az_L	azimuth angle of the launcher (measured clockwise from north)
Az_T	azimuth angle of the track (measured clockwise from north to the direction of motion of the cart)
Az_{W_A}	azimuth angle of the ambient wind (measured clockwise from north to the direction from which the wind blows)
(a_x, a_y, a_z)	components of the earth's angular velocity presented in the earth-fixed reference system
(C_1, C_2, C_3)	coefficients for the quadratic equation used to compute parachute deployment delay time
(d_1, d_2, d_3)	diameters of the first, second, and third stage configurations, respectively
$\lvert\vec{F}\rvert$	total external force exerted on the rocket
(f_1, f_2)	as subscripts denote, respectively, booster and airframe fin contributions
(F_X, F_Y, F_Z)	components of $\lvert\vec{F}\rvert$ along the missile fixed X,Y,Z axes, respectively
g	acceleration due to gravity
I_X	instantaneous axial moment of inertia of the rocket
I_Y	instantaneous transverse moment of inertia of the rocket
k^2	square of the radius of gyration of the rocket about the longitudinal axis of the rocket

GLOSSARY (Continued)

k_p^2	square of the radius of gyration of the propellant about a transverse axis through the propellant center of gravity
ℓ	length associated with jet damping moment
L	length of the missile
$\bar{L}$	latitude of the launcher
m	instantaneous mass of the rocket and propellant
m_p	instantaneous mass of the propellant
0	as a subscript denotes time of propellant ignition
(p, q, r)	components of the instantaneous angular velocity of the missile with respect to the inertial reference axes, measured about the missile fixed (X,Y,Z) axes, respectively
R	length of launcher
r_e	radius of the earth
r_N	perpendicular distance between the malaligned thrust vector and the missile center of gravity position
S	maximum cross sectional area of the missile
t	time from ignition
$t_{V_{max}}$	time maximum velocity is reached
$t_{3,0}$	time of full deployment of the parachute
Δt_{1-2}	time between motor separation and time of maximum velocity
Δt_{2-3}	time between separation of the airframe from the payload and full deployment of the parachute
T	thrust

GLOSSARY (Continued)

$T^*(y)$	temperature of the air, function of altitude, (y)
(u, v, w)	components of the instantaneous translational velocity of the missile center of gravity with respect to the earth, presented along the X,Y,Z axes, respectively
(u_a, v_a, w_a)	components of the instantaneous translational velocity of the missile center of gravity with respect to the air, presented along the X,Y,Z axes, respectively
V	magnitude of the instantaneous translational velocity vector of the missile with respect to the earth
V_a	magnitude of the instantaneous velocity of the missile with respect to the air
V_{accel}	integral, with respect to time, of the gravity free acceleration of the missile fixed accelerometer
V_g	average velocity of the jet gases
W_A	magnitude of the velocity vector of the ambient wind with respect to the earth
W_c	cart speed, relative to the earth
(W_X, W_Y, W_Z)	components of the wind vector resolved along the X,Y,Z axes, respectively
(W_x, W_y, W_z)	components of the wind vector resolved along the x,y,z axes, respectively
(x,y,z)	earth-fixed reference frame (an orthogonal, right-handed system of axes having x horizontal and pointing down range, y vertically upwards, and z perpendicular to the xy plane)
(X,Y,Z)	missile axis reference system (an orthogonal, right-handed system of axes with X coinciding with the longitudinal axis of the rocket; Y is in the vertical plane and perpendicular to X; Z is constrained to lie in the horizontal (xz) plane

GLOSSARY (Continued)

α	yaw angle
α_1, α_2	input constants used to generate the instantaneous center of gravity position and the instantaneous transverse moment of inertia of the missile
δ	angle of trim (angle between the missile X axis and the aerodynamic trim axis, the latter being a missile fixed axis which defines the direction of the airspeed vector for which there are no aerodynamic normal forces)
δ_A	fin cant angle (dihedral angle between the plane of the fin and the plane containing the longitudinal axis of the missile and the center of the fin)
ϵ	angle between the missile X axis and the thrust vector
θ	angle of the missile longitudinal axis with respect to the horizontal (xz) plane (positive toward the positive y axis)
$\varkappa$	thrust axial moment coefficient
λ_g	distance between the nose and the instantaneous center of gravity position of the missile
λ_{accel}	distance between the nose of the missile and the center of gravity of the accelerometer
λ_p	distance between the nose of the missile and the center of gravity of the propellant
$\lambda_{Nb}(\alpha)$	distance between the nose of the missile and the center of pressure of the normal force for the "body minus fins" configuration of the missile
λ_{N1}, λ_{N2}	distance between the nose of the missile and the centers of pressure of the normal forces generated on the booster and airframe fins, respectively
Λ	the dihedral angle between the XZ plane and the yaw plane

GLOSSARY (Continued)

$\mu_1(t)$, $\mu_2(t)$	time dependent scaling factors for side and restoring moment coefficients generated by the booster and airframe fins, respectively
$\rho_{(y)}$	density of the air (function of altitude, y)
ϕ	missile roll angle, measured about the missile X axis
ϕ_c	dihedral angle between the plane formed by the thrust vector and the missile axis and a vertical plane, parallel to the vertical plane containing the missile longitudinal axis, through the center of the nozzle base
ϕ_N	dihedral angle between the vertical plane containing the missile longitudinal axis and the plane formed by the missile longitudinal axis and the malaligned thrust vector
ϕ_{th}	instantaneous dihedral angle between the missile Y axis and the plane containing the malaligned thrust vector and a vector parallel to the missile X axis; positive when measured clockwise from the vertical plane
ϕ_{tr}	instantaneous dihedral angle between the missile Y axis and the plane containing the aerodynamic trim axis and a vector parallel to the missile X axis; positive when measured clockwise from the vertical plane
ψ	azimuth angle of the missile measured in the horizontal plane relative to the x axis (positive about the negative y axis)
ψ_W	angle between the horizontal wind component and the x axis (positive about the negative y axis)
ω	total angular velocity of the missile with respect to the earth
Ω	angular velocity of the earth about the polar axis

GLOSSARY (Continued)

$C_{\ell p}(M)$	spin damping moment coefficient (function of Mach number)
$C_{\ell \delta_A}(M)$	spin moment coefficient due to cant angle of the fins (function of Mach number)
$C_{Nb}(M,\alpha)$	Normal force coefficient for the configuration without fins (function of Mach number angle of attack)
$C_{np}(M,\alpha)$	magnus moment coefficient (function of Mach number and angle of attack)
$C_{Nof_1}(\alpha)$	nonroll-dependent component of the component of the total normal force on the booster fins (function of yaw)
$C_{Nof_2}(\alpha)$	nonroll-dependent component of the component of the total normal force coefficient on the airframe fins (function of yaw)
$C_{Nsf_1}(\alpha)$	roll-dependent component of the component of the total normal force on the booster fins (function of yaw)
$C_{Nsf_2}(\alpha)$	roll dependent component of the component of the total normal force on the airframe fins (function of yaw)
$C_{Y7f_1}(\alpha)$	side force coefficient, booster fin component (function of yaw)
$C_{Y7f_2}(\alpha)$	side force coefficient, airframe fin component (function of yaw)
$C_{mq}(M,\alpha)$	cross spin damping moment coefficient (function of Mach number and yaw)
$C_X(M,\alpha)$	axial drag force coefficient for the first stage configuration (function of Mach number and yaw)
$C_{X2}(M)$	axial drag force coefficient for the second stage configuration (function of Mach number)

GLOSSARY (Continued)

$C_{X3}(M)$	axial drag coefficient for the third stage configuration (function of Mach number)
$C_{yp}(M,\alpha)$	magnus force coefficient (function of Mach number and angle of attack)
$C_{Z0}(M,\alpha)$	nonroll-dependent component of the total normal force coefficient (function of Mach number and yaw)
$C_{Z8}(M,\alpha)$	roll-dependent component of the total normal force coefficient (function of Mach number and yaw)

APPENDIX B

DERIVATION OF ANGULAR VELOCITY COMPONENTS OF THE MISSILE RELATED TO THE INERTIAL REFERENCE FRAME

Since the Z axis of the missile-fixed reference frame is constrained to lie in a horizontal plane, consideration must be given to the relationship between the angular velocity components of the missile and the angular velocity of the coordinate system. Referring to the figure below, if we let e_1, e_2, and e_3 be unit vectors along the X, Y, and Z axes, respectively, the angular velocity of the e_1, e_2, e_3 system will have the component $\dot{\theta}$ about the e_3 and $-\dot{\psi}$ about the upward vertical, e_2. Let Ω = angular velocity of e_1, e_2, e_3 with respect to the x,y,z axes. Then

$$\Omega = \dot{\theta}\, e_3 - \dot{\psi}\, e_2 \quad .$$

Resolved in the instantaneous directions of the unit vectors e_1, e_2, e_3, Ω is:

$$\Omega = \begin{pmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{pmatrix} = \begin{pmatrix} \dot{\psi} \sin\theta \\ -\dot{\psi}\cos\theta \\ \dot{\theta} \end{pmatrix}$$

Now, let ω be the total angular velocity of the rocket such that, in component form

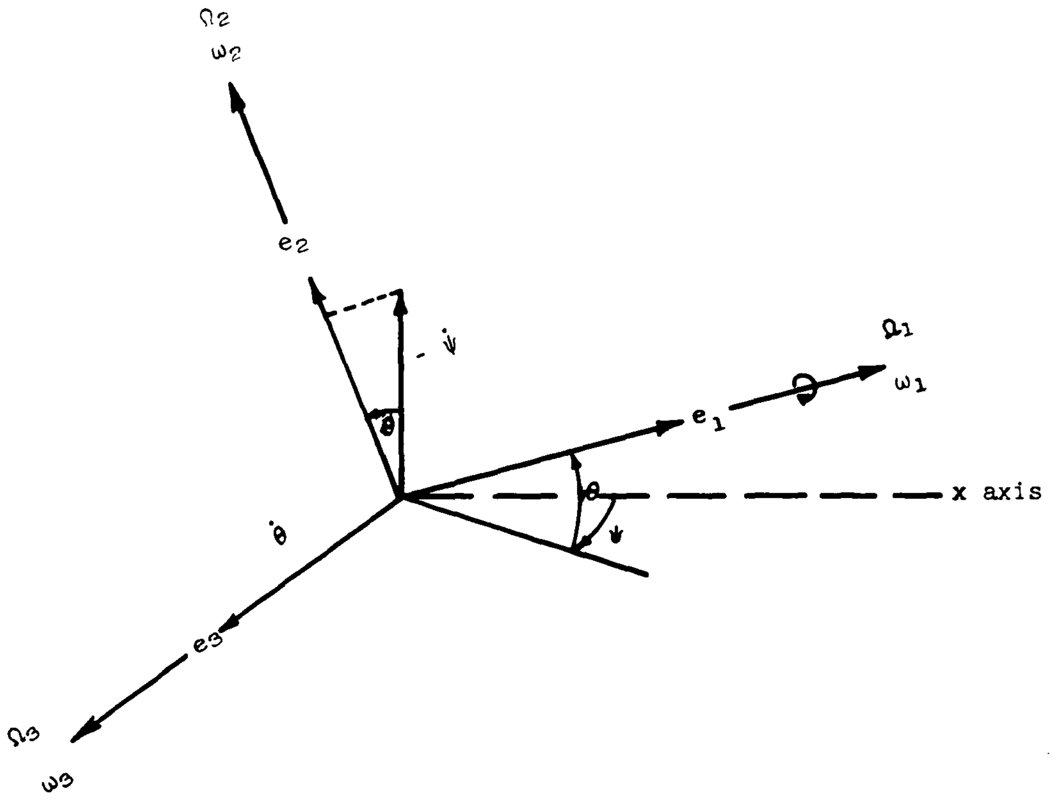
$$\omega = \begin{pmatrix} p \\ q \\ r \end{pmatrix}$$

Since e_1, e_2, e_3 do not turn about e_1, its angular velocity is independent of the p component. Because e_3 is always in a horizontal plane q and Ω_2 are the same about the y and e_2 axes, respectively, as are the angular velocities r and Ω_3 about the e_3 axis. That is,

$$\Omega_2 = q$$

$$\Omega_3 = r \quad .$$

RELATION BETWEEN THE ANGULAR VELOCITY OF THE MOVING e_1, e_2, e_3 SYSTEM AND THE ANGULAR VELOCITY OF THE ROCKET



$$\Omega_1 = -\dot{\psi}\sin\theta = \omega_2\tan\theta$$

$$\Omega_2 = -\dot{\psi}\cos\theta = \omega_2$$

$$\Omega_3 = \dot{\theta} = \omega_3$$

Further

$$\begin{aligned}\Omega_1 &= -q \sin \theta \\ &= (-q/\cos \theta) \sin \theta \\ &= -q \tan \theta \ .\end{aligned}$$

Therefore,

$$\begin{pmatrix} \dot{\phi} \\ \dot{\psi} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} p - q \tan \theta \\ -q/\cos \theta \\ r \end{pmatrix} .$$

Note that elsewhere in the report Ω represents the angular rate of rotation of the earth about its polar axis.

APPENDIX C

INPUT PARAMETERS AND INITIAL CONDITIONS REQUIRED FOR TRAJECTORY COMPUTATIONS ON THE IBM 7090 COMPUTER

Exponential Input (Format 5E 14.7)

t_o (sec)	motor ignition time
m_o (slugs)	mass at motor ignition time
$-R$ (ft)	negative of the launcher length
g (ft/sec^2)	gravitational acceleration
θ (deg)	launcher elevation angle
Az_{W_A} (deg)	azimuth angle, with respect to north, of the wind (measured from north to the direction from which the wind comes)
W_A (ft/sec)	wind velocity when constant winds for all altitudes are assumed
$\left.\begin{matrix}\Delta u_L \\ \Delta v_L \text{ (ft/sec)} \\ \Delta w_L\end{matrix}\right\}$	incremental velocity components introduced at time of launch
$\left.\begin{matrix}p_L \\ q_L \text{ (rad/sec)} \\ r_L\end{matrix}\right\}$	missile angular velocity components at launch
V_g (ft/sec)	effective gas velocity
constant	scaling factor for thrust
t (sec) or V (ft/sec)	value sensed to stop computations in the first stage
constant	scaling factor for air density
ϕ_L (deg)	roll angle of the missile fins

ψ_L (deg)	azimuth angle of the missile at launch
y_o (ft)	altitude at ignition
control word for wind	if zero, the first wind table is used; if nonzero, the next wind table is used
control word for air temperature and air density	if zero, the first air temperature and air density table is used; if nonzero, the next air temperature and air density table is used
Az_T (deg)	azimuth angle of the track (measured from north to the direction of motion of the cart)
Az_L (deg)	azimuth angle of the launcher with respect to north
ℓ (ft)	length associated with jet damping
Ω (rad/sec)	angular rate of rotation of the earth about its polar axis
I_{XB} (slug-ft^2)	missile axial moment of inertia when all propellant is expended
t (sec)	time when the scaling factors for the restoring moment coefficient, $\mu_{1,2}(t)$, equal unity
Δt (sec)	integration interval used between time where $\mu_{1,2}(t) = 1$ and the end of the first stage
L (ft)	length of the missile
λ_p (ft)	distance from the nose of the missile to the center of gravity of the propellant
λ_{accel} (ft)	distance from the nose of the missile to the center of gravity of the accelerometer

k^2 (ft^2)	radius of gyration squared
$1/r_e$ (ft^{-1})	reciprocal of the radius of the earth
$\bar{L}$ (deg)	latitude of the launcher
W_c (ft/sec)	cart velocity (used when simulating firings from a moving cart)
ϵ (deg)	angle between the malaligned thrust vector and the missile longitudinal axis
ϕ_N (deg)	orientation with respect to the vertical plane containing the launching lugs of the plane containing r_N
$\phi_{th,o}$ (deg)	orientation of the plane containing ϵ
$\phi_{tr,o}$ (deg)	orientation of the plane containing δ
δ (deg)	aerodynamic trim angle
κ (ft)	length associated with thrust axial moment
α_1 (slug-ft)	constant used to define the instantaneous center of gravity position and the transverse moment of inertia
α_2 (slug-ft^2)	constant used to define the instantaneous transverse moment of inertia
m_{po} (slugs)	mass of the propellant at ignition
Δt_L (sec)	integration interval to be used when simulating motion on the launcher
Δt_1 (sec)	integration interval to be used when simulating motion during first stage
δ_A (rad)	cant angle of the fins
d_1 (ft)	diameter of the first stage

ΔT^*	addend used to convert temperature from unit used in the table to °C
linear unit used per foot	conversion factor to convert linear and velocity units given in the input to ft and ft/sec
angular unit used per radian	conversion factor to convert angular units given in input to rad and rad/sec
angular unit used per degree	conversion factor to convert angular unit used in computations to deg and deg/sec
k_p^2 (ft^2)	square of the radius of gyration of the propellant
t_a (sec)	time at which the accelerometer starts performing
constant	scaling factor for C_{yp}
r_N (ft)	perpendicular distance between the malaligned thrust vector and the missile center of gravity position
ϕ_c (deg)	dihedral angle between the plane formed by the thrust vector and the missile axis and a vertical plane, parallel to the vertical plane containing the missile longitudinal axis, through the center of the nozzle base
constant	scaling factor for C_{NOf_1}
constant	scaling factor for C_{NOf_2}
constant	scaling factor for C_{N8f_1}
constant	scaling factor for C_{N8f_2}
constant	scaling factor for C_{Y7f_1}
constant	scaling factor for C_{Y7f_2}
constant	scaling factor for C_{mq}

constant	scaling factor for $\mu_1(t)$
constant	scaling factor for $\mu_2(t)$
constant	scaling factor for C_{Z0}
constant	scaling factor for $C_{Z\delta}$
constant	scaling factor for C_{Nb}
constant	scaling factor for $C_{\ell P}$
constant	scaling factor for $C_{\ell\delta_A}$
constant	scaling factor for C_{X_1}
λ_{g2} (ft)	distance from the nose to the center of gravity of the second stage missile configuration
I_{X_2} (slug-ft^2)	axial moment of inertia of the second stage configuration
I_{Y_2} (slug-ft^2)	transverse moment of inertia for the second stage configuration
m_2 (slugs)	mass of the second stage configuration
d_2 (ft)	diameter of the second stage configuration
t_{2E} (sec)	time when separation of the airframe is initiated
$\Delta\dot{x}_2$ (ft/sec)	horizontal component of impulsive velocity imparted to the second stage
$\Delta\dot{y}_2$ (ft/sec)	vertical component of impulsive velocity imparted to the second stage
$\Delta\dot{z}_2$ (ft/sec)	lateral component of impulsive velocity imparted to the second stage

Δt_{1E} (sec)	time between motor separation and the time maximum missile velocity is attained
t_2 (sec)	time maximum velocity is attained (used only when starting a trajectory in the second stage)
x_2 (ft), y_2 (ft), z_2 (ft)	position coordinates of the second stage (used only when starting a trajectory in the second stage)
$\dot{x}_2$ (ft/sec), $\dot{y}_2$ (ft/sec), $\dot{z}_2$ (ft/sec)	components of velocity for the second stage (used only when starting a trajectory in the second stage)
t_{2E1} (sec), t_{2E2} (sec)	bracketing times used to determine the time of parachute deployment which would minimize third stage flight time while satisfying a critical water entry velocity
Δt_2 (sec)	integration interval to be used when simulating motion during the second stage (particle model)
constant	scaling factor for the second stage drag coefficient
m_3 (slugs)	mass of the third stage configuration
d_3 (ft)	diameter of the third stage configuration
y (ft)	descending altitude on which to stop the computations
constant	scaling factor for the third stage drag coefficient, $C_{X3}(M)$

C_1 (sec), C_2 (sec^2/ft), C_3 (sec^3/ft^2)	constants for the quadratic equation used to evaluate Δt_{2-3}, the parachute delay deployment time
t_3 (sec)	time when the parachute is fully deployed (used only when starting a trajectory in the third stage)
x_3 (ft), y_3 (ft), z_3 (ft)	position coordinates when the parachute is fully deployed (used only when starting a trajectory in the third stage)
$\dot{x}_3$ (ft/sec), $\dot{y}_3$ (ft/sec), $\dot{z}_3$ (ft/sec)	components of velocity when the parachute is fully deployed (used only when starting a trajectory in the third stage)
Δt_3 (sec)	integration interval to be used when simulating motion during the third stage
V_{WE} (ft/sec)	water entry velocity (use zero except when running the special program to minimize third stage flight time with the parachute deployed)
ΔV_{WE}	tolerance on the water entry velocity above
θ_2 (deg)	elevation angle of the missile at the beginning of the second stage
ψ_2 (deg)	azimuth angle of the missile at the beginning of the second stage

Δp_2 (rad/sec), Δq_2 (rad/sec), Δr_2 (rad/sec)	incremental components of missile angular velocity for the second stage (used only when starting computations in the second stage using rigid body equations)
p_2 (rad/sec), q_2 (rad/sec), r_2 (rad/sec)	components of missile angular velocity for the second stage
δ_{A_2} (rad)	cant angle of the fins of the second stage configuration
constant	scaling factor for C_{X_2}
constant	scaling factor for $C_{Z0,2}$
constant	scaling factor for $C_{Z\delta,2}$
constant	scaling factor for $C_{m\delta,2}$
constant	scaling factor for $C_{Np,2}$
constant	scaling factor for $C_{m,2}$
constant	scaling factor for $C_{mq,2}$
constant	scaling factor for $C_{yp,2}$
constant	scaling factor for $C_{\ell p,2}$
constant	scaling factor for $C_{\ell\delta_{A},2}$
constant	scaling factor for $C_{y7,2}$
y_{2E}	altitude (descending) on which the second stage is terminated (option)

Integral Input (Format 10I3)

Locations	Definition of Constants
1, 2, 3	Stage to start computations If 0 - start computations at t_o If 1 - start computations at launch If 2 - start computations after motor separation If 3 - start computations after airframe separation time
4, 5, 6	Stage to stop computations If 0 - stop computations at the end of the launcher If 1 - stop computations at the time of motor separation If 2 - stop computations at airframe separation time If 3 - stop computations in the third stage at altitude given in exponential input
7, 8, 9	Item number of first trajectory to be computed
10, 11, 12	Control for wind data If 0 - winds provided in tables as f(y) will be used If 1 - constant wind given in exponential input will be used
13, 14, 15	Control for stopping thrust phase If 0 - stop on value of t given in exponential input If 1 - stop on value of V_{accel} given in exponential input

Locations	Definition of Constants
	If 2 - stop on value of V_a given in exponential input
	If 3 - stop on value of V given in exponential input
16, 17, 18	Print control (output from 6 degree of freedom model used to compute the thrust phase trajectory)
	If n - (n any number < 1000) every nth line computed will be printed
19, 20, 21	Print control (output from 6 degree of freedom model used to compute the after-burning phase of the trajectory)
	If n - (n any number < 1000) every nth line computed will be printed
22, 23, 24	Print control (output from particle model) when used to compute the afterburning phase of the trajectory
	If n - (n any number < 1000) every nth line will be printed
25, 26, 27	Select model for second stage computations
	If 0 - six degree of freedom model
	If 1 - particle ballistic model
28, 29, 30	Stop control (second stage)
	If 0 - second stage is terminated on input value of t_{2E-1}, then t_{2E-2} and enter a determination of t_{2E}, $t_{2E-1} < t_{2E} < t_{2E-2}$ such that third stage flight time is minimized
	If 1 - second stage is terminated on input value of t_{2E}
	If 2 - second stage is terminated on input value of altitude, y_{2E}

Specifications for Table Sizes and Formats

The coding permits flexibility in grid sizes of tables as given below. Table sizes must be consistent with specifications given on cards containing table controls. The controls are as follows:

In the format 14I4 list

a. The number of Mach numbers and angles of attack used to form the aerodynamic tables $C_X(M,\alpha)$, $C_{Z0}(M,\alpha)$, $C_{Z8}(M,\alpha)$, $C_{Nb}(M,\alpha)$

b. The number of Mach numbers used in the total given in "a." above

c. The number of angles of attack used in the total given in "a." above

d. The number of Mach numbers and angles of attack used to form the $C_{mq}(M,\alpha)$ table

e. The number of Mach numbers included in total given in "d." above

f. The number of angles of attack included in the total given in "d." above

g. The number of Mach numbers and angles of attack used to form the $C_{yp}(M,\alpha)$ table

h. The number of Mach numbers included in the total given in "g." above

i. The number of angles of attack included in the total given in "g." above

j. The number of Mach numbers and corresponding $C_{\ell_p}(M)$ values

k. The number of Mach numbers included in the total given in "j." above

l. The number of Mach numbers and corresponding $C_{\ell\delta_A}(M)$ values

m. The number of Mach numbers included in the total given in "l." above

n. The number of angles of attack plus the total number of data points in tables C_{NOf_1}, $C_{N\delta f_1}$, C_{NOf_2}, $C_{N\delta f_2}$, C_{y7f_1}, C_{y7f_2}, λ_{Nb}, λ_{yp}, λ_{N1}, and λ_{N2}, all assumed to be functions of angle of attack only

o. The number of angles of attack included in the total shown in "n." above

p. The number of Mach numbers and angles of attack used to form the aerodynamic tables $C_{X,2}$, $C_{Z0,2}$, $C_{Z\delta,2}$, $C_{m,2}$, $C_{m\delta,2}$, and $C_{np,2}$, all assumed as functions of Mach number and angle of attack

q. The number of Mach numbers included in the total given in "p." above

r. The number of angles of attack included in the total given in "p." above

s. Same as "d." except that $C_{mq}(M,\alpha)$ applies to the second stage rigid body configuration

t. Same as "e." except $C_{mq}(M,\alpha)$ applies to the second stage rigid body configuration

u. Same as "f." except $C_{yp}(M,\alpha)$ applies to the second stage rigid body configuration

v. Same as "g." except $C_{\ell p}(M)$ applies to the second stage rigid body configuration

w. Same as "h." except $C_{\ell\delta_A}(M)$ applies to the second stage rigid body configuration

x. The total number of angles of attack and corresponding C_{y7} values

y. The number of Mach numbers included in the total given in "x." above

z. The total number of Mach numbers and corresponding $C_{X,2}(M)$ values

aa. The number of Mach numbers included in the total shown in "z." above

bb. The total number of Mach numbers and corresponding $C_{X,3}(M)$ values

cc. The number of Mach numbers included in the total shown in "bb." above

dd. The number of values of time and corresponding thrust

ee. The number of values required to specify the wind vector as functions of altitude for the ascending and descending branches of the trajectory

ff. The number of values of time and corresponding $\mu_1(t)$

gg. The number of values of time and corresponding $\mu_2(t)$

Items "a." through "o." apply to the first stage, "p." through "y." the second stage rigid body trajectory, "z." through "bb." the second stage particle trajectory, and "cc." the third stage particle trajectory. Items "dd.", "ff." and "gg." apply to the first stage rigid body trajectory only. Item "ee." applies to all stages.

The format for all tables of aerodynamic data is 8F9.3. Formats for other tables provided for are:

Thrust	8F9.3	(time, thrust, etc.)
Wind	5F14.2	(altitude, wind azimuth, wind magnitude (ascending) and wind azimuth, wind magnitude (descending))
Air density and temperature	2F12.2 F12.5 2F12.2 F12.5	(altitude, air temperature, air density, altitude, air temperature, air density, etc.)

Input Generator

Input for as many trajectories as desired may be generated from a basic set of input by using the input generator. One IBM card in the format 24I3 indicates any one of five types of input generation allowed, the number of changes to be made, etc., as described below.

Let N1 - - - N8 be the eight words appearing on the input generating card; then,

a. If N1 = 1, i.e., the first word is 001,

N2 gives the number of complete sets of input to be generated

N3 gives the number of elements in the exponential input format to be changed

N4 gives the number of elements in the integral input format to be changed

N5 is 000 or 001 depending on whether or not the input for the basic trajectory or input from the immediately preceding trajectory, respectively, is to be changed

N6 and N7 are zero

N8 is 000 or 001 depending on whether or not computations should cease or more input follows, respectively.

If N2 is not zero the card(s) following must contain words of format 4(I3, E14.7) which specify the element number in the basic trajectory to be changed followed by the new value for the input quantity. If N3 is not zero the cards of format 4(I3, E14.7) are followed by card(s) of format 24I3 which contain the element number of the integral input for the basic trajectory which is to be changed followed by the new value, etc.

b. If N1 = 2, this signifies that a single element is to be changed from the value given in the basic input

N2 gives the element number of the input quantity in the basic exponential input format to be changed

N3 gives the number of values the element should take

N4 gives the element number of the input quantity in the basic integral input format to be changed

N5 gives the number of values the element should take

N6 and N7 are zero

N8 is as defined for the case N1 = 1

If N2 is not zero the cards following the input generating card will contain as many values as are indicated by N3, in the format 5E14.7. If N4 is not zero cards following the input generating card will contain as many values as are indicated by N5, in the format 24I3.

c. If N1 = 3, this signifies that a two parameter combination of changes is to be made to the basic exponential input

N2 and N4 give the element numbers of the parameters to be changed

N3 and N5 give the numbers of values assigned to N2 and N4, respectively; values of the parameters are given in format 5E14.7

N5 is as defined for the case N1 = 1

N6 and N7 are zero

N8 is as defined for the case N1 = 1

(N2 is held constant while N3 varies)

d. If N1 = 4, this signifies a three parameter combination of changes with N6 and N7 used as described for N4 and N5 above

e. If N1 = 5, this signifies that only one trajectory is to be computed. No input generation takes place

Item numbers automatically increase by one for each succeeding trajectory generated.

Arrangement of Input Data on Tapes

Input Tape (2)

Program and subroutines

Exponential input

Program controls

$\left.\begin{array}{l}\text{Wind table(s)} \\ \text{Air density and air temperature tables}\end{array}\right\}$ if required

Special Tape (B3) (First stage (six degree of freedom model))

Controls for tables

Mach number and angle of attack grid for $C_X(M,\alpha)$, $C_{Z0}(M,\alpha)$, $C_{Z8}(M,\alpha)$, $C_{Nb}(M,\alpha)$ tables

$C_X(M,\alpha)$ table

$C_{Z0}(M,\alpha)$ table

$C_{Z8}(M,\alpha)$ table

$C_{Nb}(M,\alpha)$ table

Mach number and angle of attack grid for $C_{mq}(M,\alpha)$ table

$C_{mq}(M,\alpha)$ table

Mach number and angle of attack grid for $C_{yp}(M,\alpha)$ table

$C_{yp}(M,\alpha)$ table

Mach number grid followed by corresponding $C_{\ell_p}(M)$ values

Mach number grid followed by corresponding $C_{\ell\delta_A}(M)$ values

α grid followed by corresponding values for tables of $C_{N0f_1}(\alpha)$, $C_{N8f_1}(\alpha)$, $C_{N0f_2}(\alpha)$, $C_{N8f_2}(\alpha)$, $C_{y7f_1}(\alpha)$, $C_{y7f_2}(\alpha)$, $\lambda_{Nb}(\alpha)$, $\lambda_{yp}(\alpha)$, $\lambda_{n1}(\alpha)$, and $\lambda_{N2}(\alpha)$

Special Tape (B3) (Second stage (six degree of freedom model))

Mach number and angle of attack grid for C_X, C_{ZO}, C_{Z8}, C_{m8}, C_{np}, and C_m tables

$C_X(M,\alpha)$ table

$C_{ZO}(M,\alpha)$ table

$C_{Z8}(M,\alpha)$ table

$C_{m8}(M,\alpha)$ table

$C_{np}(M,\alpha)$ table

$C_m(M,\alpha)$ table

Mach number and angle of attack grid for $C_{mq}(M,\alpha)$ table

$C_{mq}(M,\alpha)$ table

Mach number and angle of attack grid for C_{yp} table

$C_{yp}(M,\alpha)$ table

Mach number grid followed by corresponding $C_{\ell p}(M)$ values

Mach number grid followed by corresponding $C_{\ell \delta_A}(M)$ values

Angle of attack followed by corresponding $C_{y7}(\alpha)$ values

Special Tape (B3)(Second stage (particle model))

Mach number grid followed by corresponding $C_X(M)$ values

Special Tape (B3) (Third stage (particle model))

Mach number grid followed by corresponding $C_X(M)$ values

Thrust table (t followed by corresponding value of T, etc.)

Wind table (y, Az_{W_A} ascending, W_A ascending, Az_{W_A} descending W_A descending, etc.)

Air temperature and air density table (y, T^* °C, ρ, etc.)

$\mu_1(t)$ table (t followed by corresponding value of $\mu_1(t)$, etc.)

$\mu_2(t)$ table (t followed by corresponding value of $\mu_2(t)$, etc.)

APPENDIX D

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SUBJECT ANALYSIS OF REPORT

DESCRIPTOR	CODE	DESCRIPTOR	CODE	DESCRIPTOR	CODE
Equations of motion	EQUM	Mathematics	MATH		
Flight	FLIG	Models	MODE		
Simulation	SIMU	Aerodynamics	AERD		
ASROC (missile)	ASRD	Drag	DRAG		
Trajectories	TRAJ	Coefficients	COEF		
THRUST	THRS				
RIGID	RIGI				
Bodies	Body				
Force	FORC				
Moment	MOME				
Gravity	GRAV				
Earth	EART				
Rotating	ROTA				

Naval Weapons Lab. (NWL Report No. 1812)
EQUATIONS OF MOTION FOR FLIGHT SIMULATION OF THE ASROC MISSILE, by Herman P. Caster
29 June 1962. 21,[28] p. figs. tables
UNCLASSIFIED

Equations of motion are presented for a three dimensional trajectory simulation under the assumption that, during the thrust phase, the configuration is a rigid body and has 90 degree rotational symmetry. For flight simulation during the after-burning phase equations based on particle ballistic theory are presented. Aerodynamic coefficients considered during the thrust phase are functions of Mach number, angle of attack and effective roll angle.

1. Equations of motion
2. Flight - Simulation
3. Mathematical models
I. ASROC
II. Caster, Herman P.

TASK No. RUSD-2A-000/210-1/W002A0-009.

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